MATHEMATICAL QUESTION TYPES

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Questions are of cardinal importance in mathematics and its teaching mathematics. Various classification schemes of mathematical question types exist and are in use. Most of these have little practical currency for the day-to-day practice of teaching. We discuss the development of a scheme of question types that evolved from the way teachers talk about examination and other assessments. The way teachers are beginning to use the scheme is demonstrated and it is concluded that such schemes evolving from the issues and dilemmas teachers face have high possibility to contribute towards more productive teaching and meaningful learning.

INTRODUCTION

School-based assessment is an important component of the teaching and learning accountabilities within the schooling system. Setting quality assessments and tests is not an easy task, as is evident from the Department of Basic Education's (DBE) report on the moderation of school-based (DBE, 2013:43) assessments. Consequently the DBE suggests the development of quality assessment tasks which will serve as exemplars to guide teachers for setting school-based assessments.

As a response this article provides a conceptual scheme of question types to assist teachers and other developers of assessment tasks.

From another angle, questioning plays an important role in the advancement of mathematics. Brown and Walter (1983: pp. 2-3) draws attention the primacy of questions when they relay how after hundreds of years of attempting to prove Euclid's fifth postulate, mathematicians got a handle on it by considering the question "*How can you prove the parallel postulate from the other postulates or axioms?*" (Italics in original) instead if just focusing on the proving of the fifth axiom.

Many typologies for classifying question types for school mathematics exist. Widely known and used are Bloom's revised and the SOLO taxonomies. In many instances these taxonomies are adapted to suit particular contexts and needs. For example, the scheme used for classifying question types to be included in school Mathematics examinations as per the Curriculum and Assessment Policy Statement (CAPS) is an adaptation of Bloom's revised taxonomy. The expectation is that teachers will use the level-based scheme of question types for the setting of school-based assessments.

Another characterisation of questions is classifying them as open or closed questions. Regarding open and closed questions Boaler & Brodie (2004) concluded that regardless of the teaching approach adopted by teachers these question types were present in the approach adopted by teachers. Watson and De Geest (2012: 227) proffers that the preponderance of closed questions does not inhibit improvement of learning.

Although the typologies are useful and productive for the design of examinations and other assessments, our experience with teachers is that they particularly find the typologies not user-friendly for their practice. A plausible reason is that these typologies are normally distributed through a "research discourse generated within education faculties [and other research environments]…not readily [accessable] by the majority of South African teachers". (Wright, 2013: 23). He calls for "a situationally committed mode of research discourse…Discourse 4, addressing the needs of teachers." (pp. 26-27).

We concur with this sentiment and in our quest to address needs of teachers, we developed with mathematics teachers, a scheme of question types which we contend are more easily understood by them and carries more practical currency for them.

EVOLUTION OF THE SCHEME

Our work with teachers is described in various publications (e.g. Julie, 2012; Julie 2013) Central to our work is the transformation of ideas offered by teachers, in their practice discourse, into classroom implementable modalities.

This particular scheme flowed from discussions during an in-service session on examination-setting and marking, the 2013 Annual National Assessments (ANA) and the 2013 National Senior Certificate examination for Mathematics. In particular the comment "When questions are turned around, learners find them difficult to deal with", focused our attention. "Questions turned around" has a particular meaning in teacher discourse. In the 2013 ANA Grade 9 Mathematics test, for example, question 1.4 is an example of a "turned around" question type. Its formulation is follows:

1.4 Given the expression $\frac{x-y}{3} + 4 - x^2$

Circle the letter of the <u>incorrect</u> statement

A The expression consists of 3 terms

B The coefficient of x is 1

C The coefficient of x^2 is -1

D The expression contains 2 variables

The "turned around" nature is embedded in the dominant school mathematics culture where for questions in elementary introductory algebra, the normal formulation is that learners have to write down the coefficients of the terms. A similar notion of a "turned around problem" was expressed about the statistics item in the 2013 NSC Mathematics Paper 2 as presented in the figure below.

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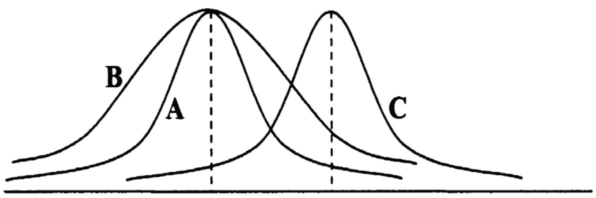
NSC

QUESTION 4

The Grade 10 classes of three schools wrote a term test. All three schools have the same number of learners in Grade 10. The results of the tests have been summarised in the table below.

	SCHOOL A	SCHOOL B	SCHOOL C
Mean	9,8	9,8	14,8
Standard deviation	2,3	3,1	2,3

The distribution of the results is shown in the diagram below.



4.1 In which school (A, B or C) is the majority of the results more widely spread around the mean? Give a reason for your answer. (2)

4.2 What is the difference in the spread around the respective means of the marks in School A and School C? (1)

4.3 Explain how the marks of School A must be adjusted to match the marks of School C. (2)

4.4 If each mark in School C is lowered by 10%, explain the effect it will have on the mean and standard deviation of this school. (2)
[7]

Normally questions dealing with this topic would require examinees to calculate the means, standards deviations without really interpreting these values. Another comment frequently made teachers is that "Children do not understand concepts". This "do not understand concepts" covers a wide range of issues. Manifestations in learner work such as slips, misconceptions, manipulative errors, incorrect application of mathematical conventions and flexible ways of understanding mathematical concepts are some of the issues.

In terms of the focus of this paper, the quest was how questions can be classified so that they focus on the above and other issues. Our approach here was to take the suggestions of Brown and Walters (1983) and Mason (2000). Brown and Walters (1983: 1) asserts “that coming to know…is to commit ourselves to…operate on…the things we are trying to understand.” Important for the developed scheme is that questions developed for learners should be such that they work with the things that their responses indicate that they do not understand.

Regarding common errors Mason (2000) offers the strategy to let learners work with examples of such errors as exposed in their work and giving them opportunity to evaluate such incorrect ways of dealing with mathematical ideas. A further strategy offered by Mason (2000: 13) is to “Ask students to make up (and do) their own questions”. This strategy can be expanded by learners setting questions for other learners and that those who set the questions (and the answers), assess the answers of those other learners.

In fact in an in-service course for mathematics teachers in the 1980’s, a teacher related how she used this strategy for setting and marking class tests. Her approach was to divide a class in groups of 5 to 6 learners. For, say, the first test group 1 would participate with her in setting and marking the class test. This procedure was followed for subsequent tests with other groups.

In response on whether learners would not leak the tests, she responded that learners were reluctant to do this since they were not certain on whether a next group will reciprocate. Further, she inculcated a strong ethic of honesty in her classes with the requisite consequences of losing marks if there is evidence tests were leaked.

Lastly,it is important to note that our deliberations in workshops rendered that a scheme must include questions that are generally commonplace in the boundary objects such as textbooks and previous examination papers.

The scheme presented below had its origin in these considerations and is explicated next.

A SCHEME OF MATHEMATICAL QUESTION TYPES

The scheme consists of five question types. These are explained and exemplified below.

STANDARD/DIRECT QUESTIONS:

These are items that are normally used as examples in teaching, which generally appear in textbooks, previous examination and other learning resource materials and with which learners have a reasonable amount of practice. Their form might vary from what was used in teaching but these variations are small. These types of questions are normally effective for practising/testing factual or procedural knowledge.

Examples:

(1) Simplify: $2(2 - 3) - 5$

(2) Solve form: $3m - 7 - 5m = 5$

(3) Complete the following: A kite is a quadrilateral with …

(4) Find the roots of $x^2 - 2x - 7 = 0$ and state what the nature of the roots are.

NON-STANDARD QUESTIONS TO CONVERT TO STANDARD FORM QUESTIONS:

The question (or part of it) is in a form that must be converted into some standard form to work towards its solution. These types of questions are especially useful for deepening knowledge and understanding of mathematics. They demand a higher level of thinking as the pathway to the solution is not directly implied in the way that the question is formulated.

Examples:

(1) Find the HCF of $2^2 \times 3 \times 5$ and 42

(2) Solve for x: $(x - 2)^2 = (x - 2) + 2$

(3) Simplify completely: $\dfrac{\sin(90^\circ + \theta) + \cos(180^\circ + \theta)\sin(-\theta)}{\sin 180^\circ - \dfrac{\sin 135^\circ}{\cos 135^\circ}}$

REVERSAL QUESTIONS:

The question is in the reverse form of the standard/direct question. In the quest for developing creativity and problem solving skills, these types of questions may be effectively used. Hence conceptual knowledge plays an important part in this type of questioning.

Examples:

(1) A problem dealing with the simplification of an expression with integers which had three different operations gave -2 as the answer. Write down the problem. Is there only one answer?

(2) Find a quadratic equation of which one root is irrational and the other a negative integer.

(3) A function f(x) was differentiated and gave the answer: $3x^5 + \dfrac{1}{2\sqrt{x}}$. Find f(x).

EVALUATIVE QUESTIONS:

These are questions which require learners to assess the correctness or not of produced answers. The most basic kind of this type of question is the true/false type. They include alternate correct ways of working and incorrect ways of working. A very good source for these kinds of questions is responses to questions in tests and examinations. This type of questioning develops meta-cognitive knowledge, in other words how one is thinking about mathematics and its procedures.

Examples:

(1) To simplify 2(2 – 3) – 5, two learners did it as follows:

Learner A	Learner B
2(2 – 3) – 5 = 2(2 – 3) – 2 – 3 = 2(2 – 3 – 1) – 3 = 2(-2) – 3 = -7	2(2 – 3) – 5 = 2(2 – 5) – 5 + 4 = 4 – 10 – 5 + 4 = 8 – 15 = -7

Are their answers correct and did they use correct methods?

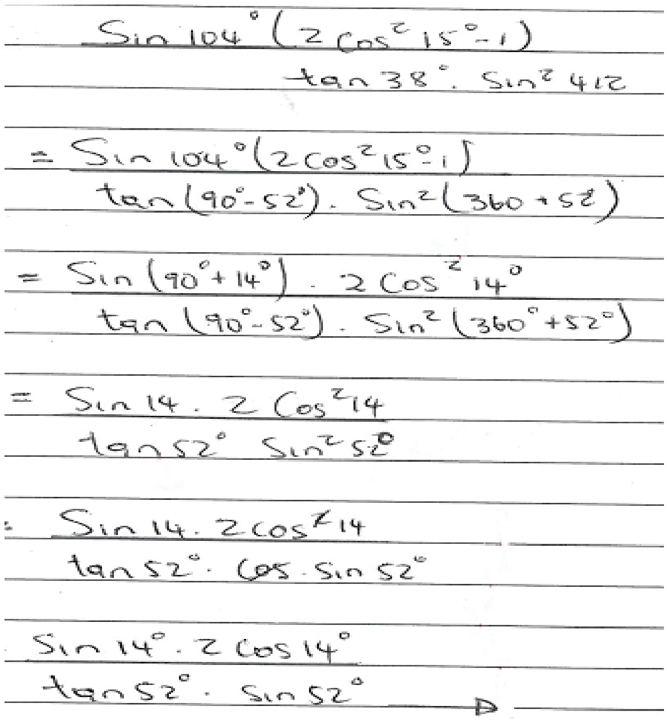
(2) To simplify $\frac{x}{x+y} - \frac{x^2 - y^2}{y^2 - x^2}$ learners A and B worked as follows:

Learner A	Learner B
$\frac{x}{x+y} - \frac{x^2-y^2}{y^2-x^2}$ $x^2+y - x^4y^4$	$x/x+y - \frac{x^2-y^2}{y^2-x^2}$ $= \frac{2x^2-y}{2x^2+2y^2}$ $= -y/2y^2 = / \frac{2xy}{4x^2y^2} = \frac{2xy}{xy}$

Do you agree with their way of working and answers?

(3) For the question: Simplify $\frac{\sin 104°(2cos^2 15°-1)}{\tan 38°.sin^2 412°}$ without the use of a calculator, a learner produced the following:

Indicate with reasons where the learner went wrong.



$$\frac{\text{Sin } 104°(2\cos^2 15°-1)}{\tan 38°. \text{ Sin}^2 412}$$

$$= \frac{\text{Sin } 104°(2\cos^2 15°-1)}{\tan(90°-52°). \text{ Sin}^2(360+52)}$$

$$= \frac{\text{Sin}(90°+14°) . 2\cos^2 14°}{\tan(90°-52°). \text{ Sin}^2(360°+52°)}$$

$$= \frac{\text{Sin } 14 . 2\cos^2 14}{\tan 52° \text{ Sin}^2 52°}$$

$$= \frac{\text{Sin } 14 . 2\cos^2 14}{\tan 52° . \cos . \text{Sin } 52°}$$

$$\frac{\text{Sin } 14° . 2\cos 14°}{\tan 52° . \text{ Sin } 52°}$$

LEARNER CONSTRUCTED QUESTIONS:

Although not strictly a question type, leaners can be asked to develop questions (and their solutions) for their peers.

Generic example:
Develop a problem (and its solution) on (a topic, mathematical construct, etc.) for the rest of the class to solve.

HOW DOES THIS SCHEME COMPARE WITH OTHER TYPOLOGIES

This question type scheme dovetails very well with the four types of knowledge as described by Krathwohl (2002, p214):

1. *Factual Knowledge* (FK): knowledge of facts, properties, definitions theorems, etc. accepted by the mathematics community
2. *Procedural Knowledge* (PK): knowledge of algotithms and accepted ways of doing mathematics, e.g. solving equations.
3. *Conceptual Knowledge* (CK): Knowledge of concepts e.g. similarity
4. *Meta-cognitive Knowledge* (MCK): Knowledge of general problem solving strategies.

The table below illustrates this alignment between the types of questions and the types of knowledge.

	Types Knowledge	Question type
1	Factual	Direct questions
2	Procedural	Direct questions
3	Conceptual	Non-standard form questions; Reversal questions
4	Metacognitive	Evaluative questions; Learner constructed questions

We, however, are of the opinion that the presented scheme due to its focus on the surface level features of the questions are more easily appropriable by teachers for use in both their teaching and the development of school-based assessments. This is illustrated in the next section.

EMERGING RESULTS OF USE OF THE SCHEME BY TEACHERS

The scheme was work-shopped in the first quarter 2014 with teachers participating in a continuing professional development initiative. As part of this workshop, the teachers had to develop questions according to the scheme for work they will do during the first quarter. In addition to standard/direct questions, teachers also generated questions of the other types.

Examples are:

Learner-generated questions:

Write a question on surds that requires the simplification of surds.

Write two questions of different difficulty level on rounding of decimal numbers.

Evaluative questions:

The following question was in a test: Simplify: $\frac{(5a)^{-2}}{5a^{-3}}$

Three learners gave the following answers

Learner 1: a

Learner 2: $5a$

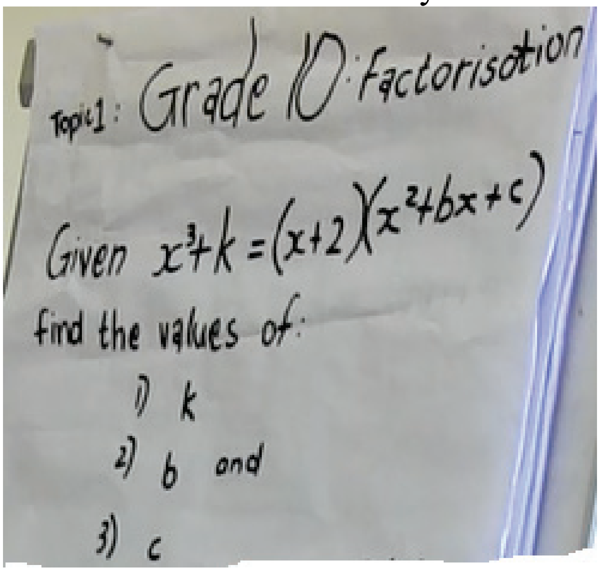
Learner 3: $\frac{a}{125}$

Which learner is correct? Show your calculations and justify your answer.
Explain the mistakes made by the other two learners.
Is $\sqrt{19}$ rounded off to one significant number equal to 4.

Non-standard form questions:

If $A = 2^n$ show that $\frac{4^{n+2}.9^{n-1}}{72^n.2^{1-n}} = 0{,}89$.

At a later workshop in the quarter teachers reported back on their use of question types in the classroom. A teacher reported that she gave her learners the question in the figure below after dealing with the factorisation of two cubes. She explained how she solved the problem and after some discussion on her way of working and possible alternatives, one of the facilitators asked: “how do you teach such problems?”



The teacher responded:

> After I did the sum and difference of cubes…I mean one of the exercises in the textbook has a lot of questions on the sum and difference of cubes. So what I did was, I went through each one and told them to analyse it. And…once we’ve factorize it, I told them look at…in your short bracket, look at your first term and in your long bracket, which was that one [points to the bracket containing the trinomial in the figure above] look at the first term of the long bracket and what is the relationship in each and every one of them. And then they said “Oh…That is that [pointing to x^2] is that one [pointing x] squared.” So I said that is what it’s gonna be in every single case. And then I told them “Look at the middle term [whilst pointing to bx] and try to find what is the relationship between these two numbers [shifting the pointing to the 2 in the first bracket]? They went through and they said “OK. It’s the same number with the same digits but the signs are different” [some faint interjection from another participant {additive inverse}]. So I told them “That is what it is gonna be in every single case. That is what they did’nt know, that it’s called additive inverse so I just told them. They have to therefore [inaudible]”. And I told them to look at this one [points to the 2 in the ‘small’ bracket] and this one [points to the c in the ‘long’ bracket] and then they said “Oh, that’s the square again.” Then I told them to look at what they starting with [points to k on the right-hand side] and look at the first bracket [pointing to the 2] and they said “Ok, Miss, it like that thingie with the three on it for the first number and the second number.” So that is how I taught it to them.

What the above report-back of a teacher illustrates is that some teachers are beginning to appropriate the scheme and using it, albeit in a limited instances, in their practice. The above is an instance of a reversal question, other teachers reported on using evaluative questions in their practice.

CONCLUSION

Schemes for classifying question types are always contestable and context-bound. What might, for example, be a reversal question for one might be a standard/direct question for another. Teachers with their intimate knowledge of their learners, their own practice and various issues that contribute towards the determination and enactment of their goals know best which question types can be classified as which for their learners.

We, however, contend that classification schemes that emerge from their concerns and dilemmas have great currency for inspiring them to extend the repertoire of question types learners are normally confronted with. This, we believe, will open ways for more productive teaching and ultimately more meaningful learning.

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